Experimental Results on Reconstruction of Complex Multidimensional Signals from Fourier Transform Magnitude

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Reconstruction of multidimensional signals from Fourier transform magnitude (FTM) has been under investigation for many years by researchers in signal processing community. Although uniqueness of reconstruction has been shown from a theoretical point of view, there seems to be no practical, stable algorithms resulting in satisfactory reconstruction of large images. Part of this numerical instability can be attributed to the fact that most real 2-D sequences with known finite support are uniquely specified to within a sign and *a time reversal*, by their FTM. This implies that the 2-D signal $f(x,y)$ and its twin image $f(-x,-y)$ have the same Fourier modulus, and therefore the iterative reconstruction algorithms are equally likely to reconstruct either one. When the support of $f(x,y)$ is symmetric, the algorithms have been shown to generate a partially reconstructed image having features of both $f(x,y)$ and $f(-x,-y)$. One way to remove this ambiguity is to introduce an additional space domain constraint which differentiates the two possible solutions. An example of such constraint which we show to result in good quality reconstruction, is specification of one bit of the amplitude of the signal in the space domain. In its most general form, one bit of space domain information can be obtained by specifying pixels above or below an arbitrary threshold.

In a more general framework, reconstruction from FTM and one bit of space domain information can be thought of as a special case of reconstruction of a complex signal from FTM and space domain phase, with the phase being equal to zero or π. Specifically, pixels above the threshold used for one bit quantization, can be thought of as having zero phase, and the ones below it can be considered to have π phase. In a similar fashion, the problem of reconstruction of real images from FTM can be considered to be a special case of reconstruction of complex signals from FTM and space domain phase, with the phase being equal to zero. Our experimental results indicate that the former reconstruction problem with binary phase, is considerably more robust than the latter one with zero phase. Thus, an interesting question to examine, from both theoretical and experimental viewpoints, would be whether reconstruction of a complex signal from FTM and space domain phase becomes better conditioned as the space domain phase becomes more "randomized". Our preliminary experiments show that reconstruction from FTM and space domain random phase with uniform probability distribution function, results in excellent reconstruction quality, and that the relative importance of FTM information with respect to Fourier transform phase information increases as the "randomness" of the phase of the complex M-D signal in space domain increases. This implies that unlike real signals, complex signals with "strong" space domain phase do not result in smooth, redundant FTM, and therefore their reconstruction from FTM and space domain phase is considerably more robust.

We discuss potential application of these results to hybrid image coding and secure image transmission schemes.