Optimal Sampling and Reconstruction of NMR Signals with Time-Varying Gradients

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Fundamental operations of nuclear magnetic resonance (NMR) imaging can be formulated, for a large number of methods, as sampling the object distribution in the Fourier spatial-frequency domain, followed by processing the digitized data to produce a digital image. In these methods, controllable gradient fields determine the points in the spatial-frequency domain which are sampled at any given time during the acquisition of the Free Induction Decay (FID) signal. Unlike the constant gradient case in which equally spaced samples of the FID signal in time correspond to uniform samples in the Fourier domain, for time-varying gradients, linear sampling in time corresponds to nonlinear sampling in the Fourier domain, and therefore straightforward inverse Fourier transformation is not sufficient for obtaining samples of the object distribution. NMR imaging methods using time-varying gradients, such as sinusoids, are particularly important from a practical point of view, since they require considerably shorter data acquisition times. In this talk, we derive an optimal sampling and reconstruction scheme for FID signals resulting from sinusoidal gradients.

Our approach is to formulate the problem as a linear parameter estimation one, by writing the observed signal, $r(t)$ as the sum of the noise-free FID signal and a zero mean, white, Gaussian random process with intensity σ^2. The noise-free FID signal is modeled to be a linear combination of the uniform samples of the object distribution, $f(n_1, n_2)$. We begin by deriving the optimal maximum likelihood (ML) estimate of $f(n_1, n_2)$ for the constant gradient case. This ML estimate is based on the observed continuous waveform $r(t)$, and our proposed discrete implementation of it, is based on uniform samples of $r(t)$. The error variance of the discrete time estimator is shown to be identical to that of the continuous time one, namely σ^2.

We then derive the optimal ML estimator based on the observed continuous waveform, $r(t)$, for the sinusoidal gradient case, and show its error variance to be approximately $1.23\sigma^2$. Two discrete implementations of the continuous time ML estimator are proposed. The first one which involves nonuniform sampling of the FID signal results in an error variance of $\frac{\pi}{2}\sigma^2$, and the second one which involves uniform sampling of the FID signal results in the same error variance as the continuous time ML estimator, provided the sampling rate is fast enough. This second discrete time implementation therefore, corresponds to the optimal sampling and reconstruction scheme for FID signals resulting from sinusoidal gradients.

The derivations for the sinusoidal gradient case are generalized to arbitrary time-varying gradients. Two major conclusions can be drawn from our results. First, uniform sampling of the FID signal in time results in the lowest possible error variance for both constant and time-varying gradients. Second, the optimal ML estimator for sinusoidal gradients has higher error variance than that of constant gradients. Future work will be directed towards the tradeoff between smoothness of the encoding gradients and their associated error variances.