Sum of Coherent Systems Decomposition by SVD

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Abstract

The Hopkins partially coherent imaging equation is expanded by eigenfunctions into a sum of coherent systems (SOCS). The result is a bank of *linear* systems whose outputs are squared, scaled and summed. This technique is useful because of the partial linearity of the resulting system approximation. The eigenfunction expansion can be accomplished by computer using the SVD algorithm. To solve this problem, the Hopkins transmission cross coefficients (TCCs) are first obtained as a matrix, then SVD is used on the matrix. Then the system is truncated at some low order (5th or 6th) to obtain an optimal approximation to Hopkins. In effect, the numerical implementation of this using SPLAT TCCs amounts to a direct approximation to SPLAT.

1 Linear Systems Approximation

The goal in this section is to compute the value of a single intensity point centered in a finite square mask region of size $L_x \times L_x$ μm^2 , as depicted in Figure 1. First, we consider the 1-D case. By periodicizing a the length L_x mask, and taking its Fourier series expansion $\tilde{G}(n)$, as done by Flanner [2] we obtain the following expression for the Fourier series of the resulting periodic intensity, $\tilde{I}(\cdot)$:

$$\tilde{I}(n) = \sum_{n'} \tilde{T}(n+n', n')\tilde{G}(n+n')G^*(n') \tag{1}$$

Suppose that the transmission cross coefficient function $\tilde{T}(n', n'')$ can be approximated by:

$$\tilde{T}(n', n'') \approx \sum_{k=1}^{N_a} \sigma_k \Phi_k(n')\Phi_k^*(n'') \tag{2}$$

for some functions $\Phi_k(\cdot), k = 1 \ldots N_a$. We call this an N_ath order approximation. Then substitute equation 2 into equation 1 to yield

$$\tilde{I}(n) = \sum_{n'} \sum_{k=1}^{N_a} \sigma_k \Phi_k(n'+n)\Phi_k^*(n')\tilde{G}(n+n')G^*(n')$$

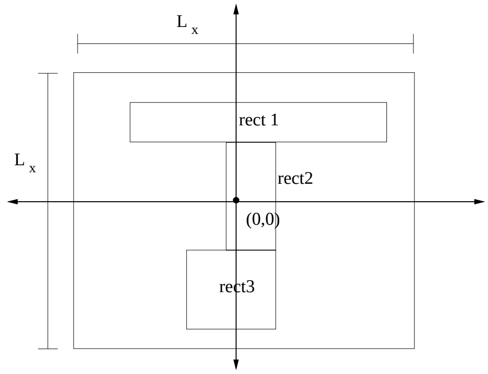


Figure 1: *Compute intensity at (0,0) for a small mask*

$$= \sum_{n'} \sum_{k=1}^{N_a} \sigma_k \Phi_k(n'+n)\tilde{G}(n+n')\Phi_k^*(n')\tilde{G}^*(n')$$

$$= \sum_{k=1}^{N_a} \sigma_k (\Phi_k \tilde{G}) \star (\Phi_k \tilde{G})^*(n) \tag{3}$$

where $\star$ is the convolution operator. This is observed to be sum of convolutions in the frequency domain. The convolutions are autocorrelations of the Fourier coefficients. By convolution-multiplication duality, the inverse Fourier series of this expression can be written in the spatial domain as

$$I(x) = \sum_{k=1}^{N_a} \sigma_k \left|(\phi_k \star g)(x)\right|^2 \tag{4}$$

In two dimensions, this is extended straightforwardly to yield the Fourier series expansion of the image intensity and the image intensity:

$$\tilde{I}(m,n) = \sum_{k=1}^{N_a} \sigma_k (\Phi_k \tilde{G}) \star (\Phi_k \tilde{G})^*(m,n) \tag{5}$$

$$I(x,y) = \sum_{k=1}^{N_a} \sigma_k \left|(\phi_k \star g)(x,y)\right|^2 \tag{6}$$

where $\star$ is now 2-D convolution. Thus, the intensity is the weighted summed squared outputs of N_a linear systems, which we call a SOCS as shown in Figure 2.

2 Decomposing Hopkin's Imaging Equation

To this point, the development of the SOCS intensity calculation technique has assumed that the original approximation in equation 2 is valid. In this section, this claim is validated and a technique for finding the 2-D convolution kernels, $\phi_k(\cdot)$, is elaborated. We start by deriving

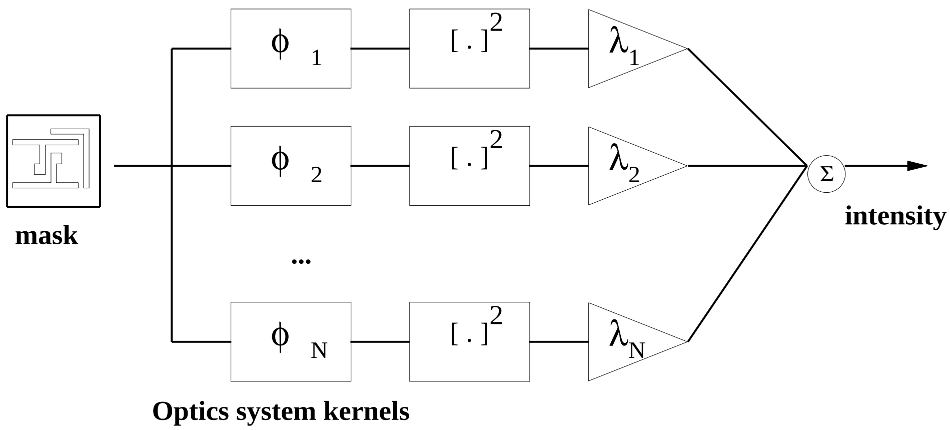


Figure 2: *Sum of coherent systems (SOCS) approximation to Hopkin's imaging*

the decomposition in the 1-D case for simplicity and then extend it naturally to 2-D. The determination of the 2-D convolution kernels requires some special techniques described in section 2.2.

2.1 1-D Decomposition

The following two facts from Flanner[2] are useful in the subsequent development.

Fact 1 $\tilde{T}(\cdot,\cdot)$ *is nonzero for only finitely many points and therefore can be represented by a matrix of values of size* $M \times M$.

Fact 2 $\tilde{T}(n', n'') = \tilde{T}^*(n'', n')$ *and therefore it is Hermitian symmetric.*

Based on this, we make the following claim:

Claim 1 $\tilde{T}(n', n'')$ *can be written as*

$$\tilde{T}(n', n'') = \sum_{k=1}^{M} \sigma_k \Phi_k(n') \Phi_k^*(n'')$$

Proof:
Putting Facts 1 and 2 together allow us to write $\tilde{T}(n', n'')$ as the Hermitian matrix $[\tilde{T}]_{i,j} = \tilde{T}(i, j)$. The dyadic expansion of T in terms of it's eigenvectors is

$$\tilde{T} = \sum_{k=1}^{M} \sigma_k \Phi_k \Phi_k^*,$$

which is equivalent to the claim. □

With this we have a sum-of-coherent systems decomposition similar to that proposed by Pati and Kaillath[3] which describes the action of the partially coherent optical system.

By truncating the summation at N_a, we get a reduced order approximation to the partially coherent system resulting in intensity:

$$\tilde{T}(n', n'') \approx \sum_{k=1}^{N_a} \sigma_k \Phi_k(n') \Phi_k^*(n'') \tag{7}$$

Since the singular values σ_k rapidly decay in magnitude, the truncation will be a good approximation. Theoretical error bounds on the approximation are analyzed in the paper by Pati and Kailath[3], in which it is shown that very good low-order approximations are achieved. The decomposition can be performed by Singular Value Decomposition (SVD) if we have the computed TCC values, $\tilde{T}$. With these results, the approximation of equation 2 and the subsequent development are fully justified.

2.2 2-D Kernel Determination

In the 2-D case, we can also use SVD to obtain the 2-D convolution kernels in the decomposition. In the 2-D case, the discrete TCCs can be thought of as a linear mapping from the space of $M \times M$ matrices, $R^{M\times M}$, to itself. So, given a matrix $X \in R^{M\times M}$, the function $\tilde{T}(i,j,k,l)$ can define a linear mapping T whose action is described by:

$$[T(X)]_{(i,j)} = \sum_{k=1}^{M} \sum_{l=1}^{M} \tilde{T}(i,j,k,l) X_{k,l}$$

This linear operation can be "unwound" to be represented as a matrix, $\mathcal{T}$, operating on a column vector from R^{N^2}. The unwinding is performed by stacking the columns of the vector and then writing the operation out as a matrix-vector multiply. The column stacking function $\mathcal{S} : R^{N\times N} \longmapsto R^{N^2}$ is defined by:

$$\mathcal{S}(X_{i,j}) = \mathcal{X}_{j*N+i}$$

For example, let X be a matrix of size $M \times M$:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1M} \\ x_{21} & x_{22} & \dots & x_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{M2} & \dots & x_{MM} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_M \end{bmatrix}$$

The column stacking operation is performed on X, yielding

$$\mathcal{X} = \mathcal{S}(X) = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{bmatrix}$$

.

The operator T defined before can be represented as a matrix, $\mathcal{T}$, operating on the stacked $\mathcal{X}$ vector,

$$\mathcal{T} = \begin{bmatrix} \tilde{T}(1,1,1,1) & \tilde{T}(1,1,2,1) & \dots & \tilde{T}(1,1,N,1) & \tilde{T}(1,1,1,2) & \tilde{T}(1,1,2,2) & \dots & \tilde{T}(1,1,N,N) \\ \tilde{T}(2,1,1,1) & \ddots & & & & & & \\ \vdots & & & & & & & \\ \tilde{T}(N,1,1,1) & & & & & & & \\ \tilde{T}(1,2,1,1) & & & & & & & \\ \tilde{T}(2,2,1,1) & & & & & & & \\ \vdots & & & & & & & \\ \tilde{T}(N,2,1,1) & & & & & & & \\ \vdots & & & & & & & \\ \tilde{T}(N,N,1,1) & \dots & & & & & & \tilde{T}(N,N,N,N) \end{bmatrix}$$

The contour plot in figure 3 shows the an example of the matrix $\mathcal{T}$. Singular value decomposition applied to this matrix yields the decomposition:

$$\mathcal{T} = \sum_{k=1}^{N} \sigma_k V_k V_k^*, \tag{8}$$

Then, the inverse column stacking operation yields the desired functions, Φ_k, in equation 2 and then it it possible to make the approximation therein:

$$\begin{aligned} \Phi_k &= \mathcal{S}^{-1}(V_k) \\ \tilde{T}(n', n'') &\approx \sum_{k=1}^{N_a} \sigma_k \Phi_k(n') \Phi_k^*(n'') \end{aligned}$$

The spatial domain convolution kernels are the Inverse Fourier Series' (IFS) of the Φ_k's. Using the 2-D Inverse Fast Fourier Transform (IFFT) to obtain the ϕ_k's yields:

$$I(x,y) = \sum_{k=1}^{N_a} \sigma_k \left|(\phi_k \star g)(x,y)\right|^2 \tag{9}$$

Using the IFFT instead of IFS introduces a small amount of destructive aliasing, but this is necessary in order to limit the time domain convolution kernels to have finite support. A plot of the singular values in figure 4 shows why a reduced order approximation can be very accurate, since the singular values quickly approach zero. The magnitudes plots of the first two convolution kernels obtained after the IFFT are shown in figure 5.

References

[1] N. Cobb, "Fast mask optimization for optical lithography," Master's thesis, University of California at Berkeley, 1994.

[2] P. D. Flanner, "Two-dimensional optical imaging for photolithography simulation," Tech. Rep. Memorandum No. UCB/ERL M86/57, Electronics Research Laboratory, University of California at Berkeley, Jul 1986.

[3] Y. C. Pati and T. Kailath, "Phase-shifting masks for microlithography: Automated design and mask requirements," *Journal of the Optical Society of America A-Optics Image Science and Vision*, vol. 11 No. 9, pp. 2438–2452, 1994.

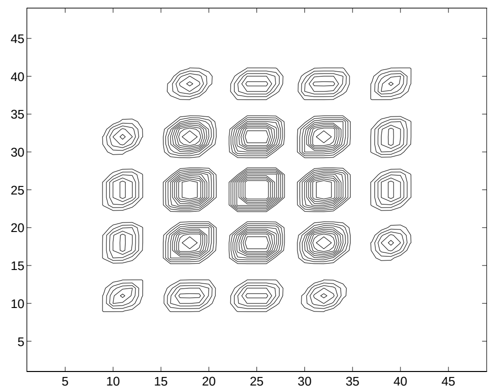


Figure 3: *Contour plot of TCCs in 2-D matrix form*

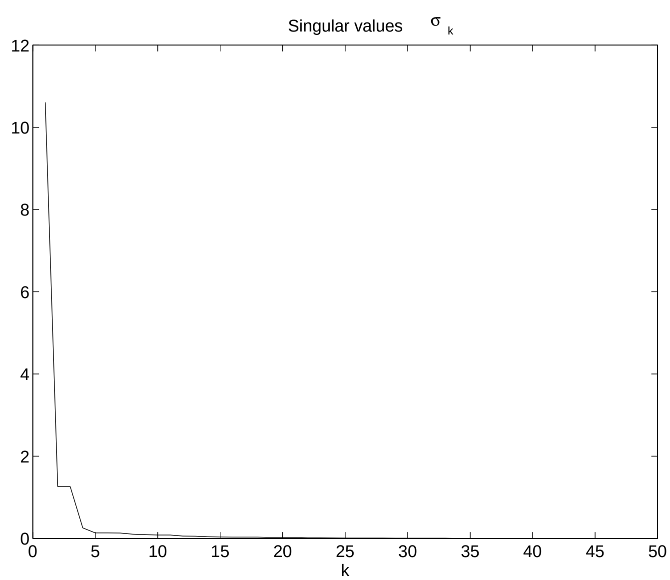
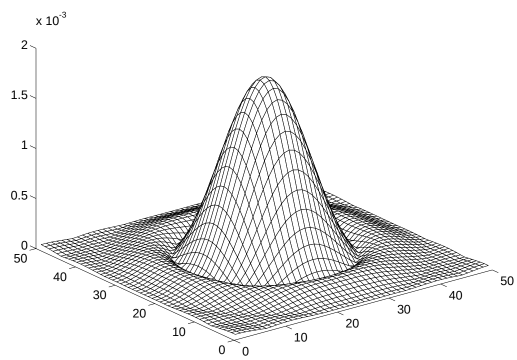
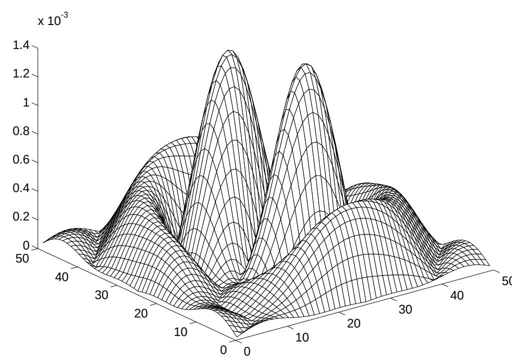


Figure 4: *Singular values, σ_k obtained in decomposition*



(a)



(b)

Figure 5: *(a)* $\phi_1(x,y)$ *(b)*$\phi_2(x,y)$